ECE 343: Signals and Systems, Exam #3, Fall 2010

Closed book, closed notes, closed homework, no calculators!
YOU MUST SHOW YOUR WORK FOR FULL CREDIT!

1. (6 pts) Mr. Kiemele likes simple problems, like $1+1$, and his signal's professor Dr. Green likes simple transforms, like the bilateral Laplace transform. To keep things doubly simple, determine $\mathcal{L}\{1+1\}$, the bilateral Laplace transform of the sum $1+1$.

$$1+1 = 2 = 2u(t) + 2u(-t) \xrightarrow{\mathcal{L}} \underbrace{\frac{2}{s}}_{\sigma>0} \cap \underbrace{-\frac{2}{s}}_{\sigma<0} \Rightarrow \text{DONE}$$

SINCE ROCS DO NOT OVERLAP, $\boxed{\mathcal{L}(1+1) \text{ DOES NOT EXIST}}$

2. Consider the signals $x(t)$ and $y(t)$, as shown in Fig. 1.

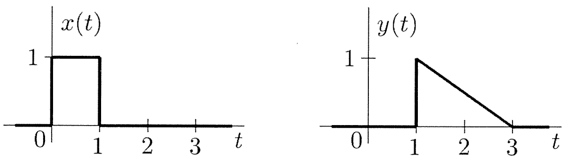


Figure 1: Signals $x(t)$ and $y(t)$ for Prob. 2.

$$= 1 - \frac{s}{2!} + \frac{s^2}{3!} - \frac{s^3}{4!} + \dots$$

(a) (8 pts) Using the definition, compute $X(s)$, the bilateral Laplace transform of $x(t)$.

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt = \int_0^1 e^{-st}\,dt = \frac{e^{-st}}{-s}\Big|_0^1 \Rightarrow \boxed{X(s) = \frac{1-e^{-s}}{s}}$$

ROC: entire s-plane {except maybe $|s| = \infty$}

Since $x(t)$ is finite duration, ROC is entire s-plane (except maybe $|s|=\infty$)

(b) (16 pts) Using Laplace transform properties, express $Y(s)$, the bilateral Laplace transform of $y(t)$, as a function of $X(s)$, the bilateral Laplace transform of $x(t)$. Simplify as much as possible WITHOUT substituting your answer from part 2a.

{MANY POSSIBLE WAYS TO OBTAIN SOLUTION}

NOTE: $y(t) = \left(\frac{3-t}{2}\right) x\left(\frac{3-t}{2}\right)$

$x(t) \xrightarrow{\mathcal{L}} X(s)$ {begin}

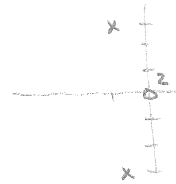
$x\left(\frac{t}{2}\right) \xrightarrow{\mathcal{L}} 2X(2s)$ {time-scale}

$x\left(-\frac{t}{2}\right) \xrightarrow{\mathcal{L}} 2X(-2s)$ {time-reverse}

$-\frac{t}{2} x\left(-\frac{t}{2}\right) \xrightarrow{\mathcal{L}} \frac{1}{\cancel{2}} \frac{d}{ds} \cancel{2} X(-2s)$ {differentiation in s}

$y(t) = -\frac{(t-3)}{2} x\left(-\frac{(t-3)}{2}\right) \xrightarrow{\mathcal{L}} e^{-3s} \frac{d}{ds} X(-2s)$ {time-shift}

$$\therefore \boxed{Y(s) = e^{-3s} \frac{d}{ds} X(-2s)\ ; \text{ ROC is entire } s\text{-plane}}$$ (like (a))



HP filter, TRANSITION & PEAK near $\omega = \pm 3$.

3. Consider an LTIC system with system function $H(s) = \frac{s^2}{(s+1+3j)(s+1-3j)}$. $= \frac{s^2}{s^2+2s+10}$

(a) (6 pts) Approximate the output of this system in response to the input $x(t) = \cos(3t) + \sin(10t)$.

@ $\omega = 3$: $|H(j3)| \approx \frac{3^2}{6(1)} = 1.5$ $\quad \angle H(j3) \approx \pi - \pi/2 = \pi/2$

@ $\omega = 10$: $|H(j10)| \approx \frac{10^2}{7(13)} = \frac{100}{91} \approx 1.1$ $\quad \angle H(j10) \approx \pi - \pi = 0$

$\therefore$ $\boxed{y(t) \approx 1.5\cos(3t + \frac{\pi}{2}) + 1.1\sin(10t)}$

ACTUAL: $y(t) = 1.4796\cos(3t + 0.5526\pi) + 1.0847\sin(10t + .0696\pi)$

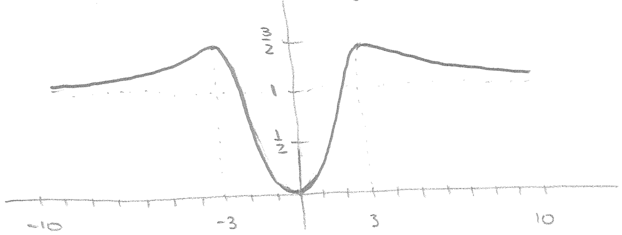
(b) (12 pts) Sketch $|H(j\omega)|$ and $\angle H(j\omega)$ over $-10 \le \omega \le 10$.

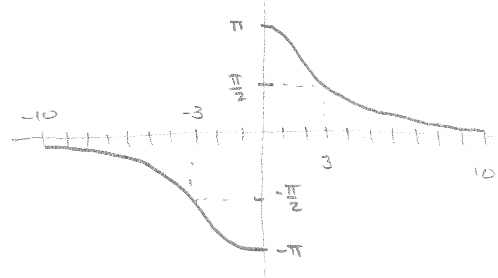
@ $\omega = 0$ $\quad |H(j0)| = 0$ $\quad$ @ 0^+ $\angle H(j0^+) = \pi$, @ 0^- $\angle H(j0^-) = -\pi$

@ $\omega = \infty$ $\quad |H(j\infty)| = 1$ $\quad \angle H(j\infty) = 0$

ALSO USE CALCULATIONS ABOVE.

even $\quad$ ODD





(c) (8 pts) Write MATLAB code that accurately plots $|H(j\omega)|$ and $\angle H(j\omega)$ over $-10 \le \omega \le 10$.

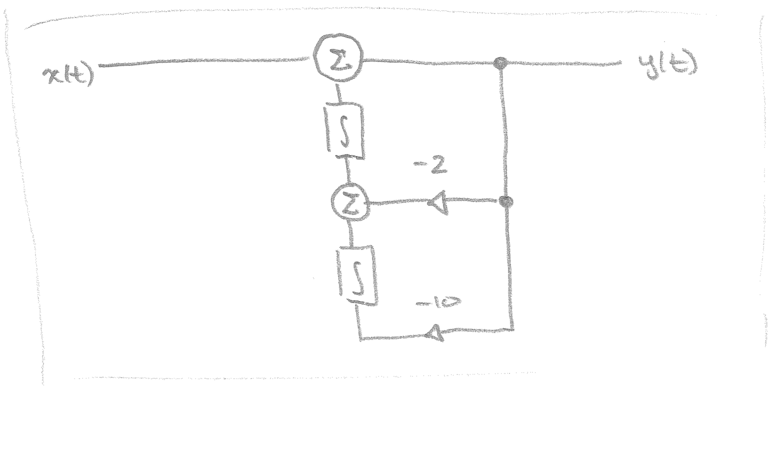
```
H = @(s) s.^2./(s.^2+2*s+10);
w = linspace(-10,10,1001);
subplot(211); plot(w, abs(H(j*w)));
subplot(212); plot(w, angle(H(j*w)));
```

SEE ATTACHED OUTPUT!

(d) (8 pts) Draw an appropriate block-diagram representation of this system.

FROM $H(s)$: $y + 2\int y + 10\iint y = x$

NORMALLY TDFII best, BUT IN THIS CASE ALL STRUCTURES ARE ABOUT THE SAME



4. Consider the op-amp circuit of Fig. 2. Further, let $RC = 1$.

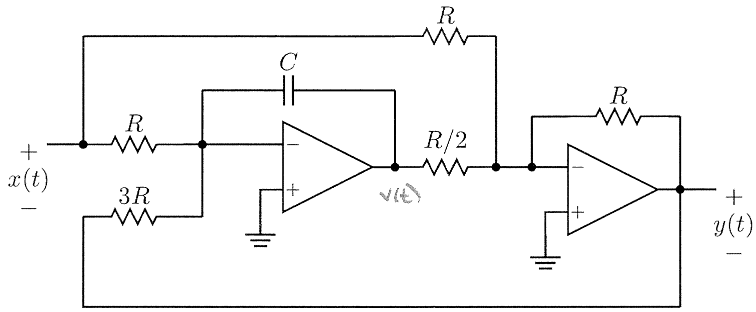


Figure 2: Op-amp circuit for Prob. 4.

(a) (12 pts) Determine the corresponding differential equation of Fig. 2. Express your simplified result in standard form.

$\frac{1}{RC}$ GAIN IS 1 ; $\frac{1}{3RC}$ GAIN IS $\frac{1}{3}$; $\frac{R}{R/2}$ GAIN IS 2

$v(t) = -\int x - \frac{1}{3}\int y$; $y(t) = -x - 2v$

$\therefore \quad y(t) = -x(t) - 2\left(-\int x(t) - \frac{1}{3}\int y(t)\right) \Rightarrow y(t) - \frac{2}{3}\int y(t) = -x(t) + 2\int x(t)$

$\therefore \quad \boxed{\dot{y}(t) - \frac{2}{3}y(t) = -\dot{x}(t) + 2x(t)}$

(b) (12 pts) Using transform-domain techniques, determine the response $y(t)$ of Fig. 2 to the input $x(t) = u(t-1)$.

TAKING $\mathcal{L}$ OF DIFF EQ IN (a)

$(s - \frac{2}{3})Y(s) = (2 - s)X(s)$ & $X(s) = e^{-s}\frac{1}{s}$

$\therefore \quad Y(s) = e^{-s}\frac{(2-s)}{s(s-2/3)} = e^{-s}\left[\frac{-3}{s} + \frac{2}{s-2/3}\right]$

since causal system & input, $\mathcal{L}^{-1}$ yields causal outputs

$\boxed{y(t) = -3u(t-1) + 2e^{2(t-1)/3}u(t-1)}$

(c) (12 pts) Using transform-domain techniques, determine the zero-input response $y_0(t)$ of Fig. 2 if the $t = 0$ capacitor voltage (first op-amp output voltage) is 2 volts.

$v(0) = 2 \Rightarrow y_0(0) = -4$

FOR ZIR:

$\dot{y}_0(t) - \frac{2}{3}y_0(t) = 0 \xrightarrow{\mathcal{L}} sY(s) - y_0(0) - \frac{2}{3}Y_0(s) = 0$

$\therefore \quad Y_0(s) = \frac{-4}{s - 2/3} \Rightarrow \boxed{y_0(t) = -4e^{2t/3}u(t)}$